eNeuro

Research Article: Methods/New Tools | Novel Tools and Methods

A how-to-model guide for Neuroscience

https://doi.org/10.1523/ENEURO.0352-19.2019

Cite as: eNeuro 2020; 10.1523/ENEURO.0352-19.2019

Received: 30 August 2019 Revised: 18 October 2019 Accepted: 25 November 2019

This Early Release article has been peer-reviewed and accepted, but has not been through the composition and copyediting processes. The final version may differ slightly in style or formatting and will contain links to any extended data.

Alerts: Sign up at www.eneuro.org/alerts to receive customized email alerts when the fully formatted version of this article is published.

Copyright © 2020 Blohm et al.

This is an open-access article distributed under the terms of the Creative Commons Attribution 4.0 International license, which permits unrestricted use, distribution and reproduction in any medium provided that the original work is properly attributed.

A how-to-model guide for Neuroscience G. Blohm¹, K.P. Kording², P.R. Schrater³ ¹Centre for Neuroscience Research, Queen's University, Kingston, ON K7L 3N6, Canada ²Departments of Bioengineering and Neuroscience, University of Pennsylvania, Philadelphia, PA 19104, U.S.A ³Departments of Psychology and Computer Science, University of Minnesota, Minneapolis 55455, U.S.A Abbreviated title: How-to-model guide Number of Figures: 5 Number of tables: 1 Number words in abstract: 95 Number words in introduction: 597 Number words in discussion: 736 Acknowledgements: We would like to thank all 300+ CoSMo participants (2011-2018) for incrementally pushing us to develop this tutorial. We would also like to acknowledge our own mentors who have been instrumental in transmitting their modeling knowledge to us, in particular, Philippe Lefèvre, Lance Optican, David Knill, Eero Simoncelli, Daniel Wolpert, Peter König, Josh Tenenbaum, and Twitter. Funding: This work was supported with funding from NIH, USA (R25; MH109110-01) and The Brain Canada Foundation, Canada. Conflicts of interest: none Corresponding author: Gunnar BLOHM (gunnar.blohm@queensu.ca; +1-613-533-3385) Abstract: Within neuroscience, models have many roles, including driving hypotheses, making assumptions explicit, synthesizing knowledge, making experimental predictions, and facilitating applications to medicine. While specific modeling techniques are often taught, the process of constructing models for a given phenomenon or question is generally left opaque. Here, informed by guiding many students through modeling exercises at our CoSMo summer school we provide a practical ten step breakdown of the modeling process. This approach

makes choices and criteria more explicit and replicable. Experiment design has long beentaught in neuroscience; the modeling process should receive the same attention.

39 40

41

1

2 3 4

5

6

7

8

9 10 11

12 13

14

15

16

17

18 19

20

21

22

23

24

25

26

27

32

33

34

35

36

Significance: Modeling in Neuroscience is often perceived as a mysterious process and is hard to teach. Here we provide the first how-to-model guide that breaks down the modeling endeavor into a step-by-step process.

45 Introduction:

56

46 The development of models is an integral part of neuroscience and related disciplines, such 47 as psychology, kinesiology and cognitive science. Models can provide unique and useful 48 insights. For example, computational models are used to compactly describe large amounts of 49 data. Models are often employed to obtain causal claims about the relation between neural 50 properties and behavior. They make predictions and can thus allow more targeted experiments. Models allow virtual experimentation making it easier to get intuitions. Models also force 51 52 scientists to make their assumptions explicit which makes scientific communication more 53 precise. Finally, models can lead to applications across science, healthcare and technology; 54 e.g. one can plan interventions by simulating their impact on brain and behavior. Model-driven 55 approaches thus accelerate progress across clinical and basic research.

57 There are countless models in neuroscience and for each modeling technique we can find a 58 paper describing how it is constructed. For the more popular techniques we can find textbooks 59 that describe the mechanics of constructing and testing models, pitfalls, tips and tricks usually 60 tailored to the particular types of data and questions that made the technique popular. 61 However, when approaching new questions, new data types, or different scientific goals and 62 objectives, it is unclear how to start. Confronted with a phenomena and a scientific goal, every 63 researcher is faced with a difficult set of questions. Which concepts should we use? Which 64 mathematical framework, i.e. technique? Which code? What should the overall logic be? All 65 these questions are currently unarticulated and hidden in the scientific training process, and 66 students implicitly learn approaches across neuroscience through imitation and mentoring. 67 While this can be an effective way of transmitting modeling techniques for ongoing questions, it 68 is an ineffective way to train students to innovate, competently address new problems, or 69 synthesize and extend methods. Instead there should be a clearly structured thought process 70 that clearly identifies how the phenomenon along with the goals of modeling give rise to the 71 ultimate models. What is missing is a procedure by which we can address a phenomenon with 72 modeling in a way that brings us closer to our scientific goals. 73

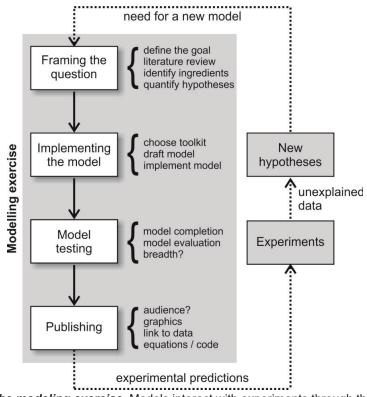
We have observed many students learning how to build models during our 8-year experience
 with CoSMo (summer school in Computational Sensory-Motor neuroscience,

76 www.compneurosci.com/CoSMo) where we taught students from senior undergraduates to 77 seasoned researchers how to model. Through teaching and project work we have tried to 78 convey to them the process of constructing models from scratch. All three of the authors are 79 also building models, and importantly we cover a broad range of types of modeling. This 80 includes machine learning, Bayesian modeling, linear systems modeling, realistic muscle 81 modeling, spiking neural networks, and single-cell models. In addition, we have brought dozens 82 of leading computational neuroscientists as guest lecturers in the course, providing us with 83 template examples of a broad array of successful modeling approaches applied to a diverse set 84 of phenomena and questions. As such, we feel that we have experienced the model 85 construction process in a uniquely crosscutting way. While the modeling process is complex and 86 multifaceted, we believe it can be formalized and made explicit.

88 Here, we propose a pipeline to modeling that breaks the whole enterprise down into a series

- 89 of (sometimes interdependent) decision processes. Note that the approach outlined here is not
- 90 the only way to approach the modeling exercise; rather it represents one possible systematic,
- 91 step-by-step approach that if carried out carefully should leave little room for failure. By
- using this approach, we have directed hundreds of CoSMo students in small groups through the
 full "from scratch" modeling process to successful conclusions in just two weeks.
- 94

102 103



95
96
Figure 1: The modeling exercise. Models interact with experiments through the generation
97
97 of novel model-based experimental predictions. Experimental data will in turn provide new
98 unexplained data and hypotheses that call for new or refined models. Note that modelers do not
99 necessarily need to test their own experimental predictions or collect their own unexplained
100 data; but good modelers should interact with experimentalists. Many good experiments come
101 from modelers annoyingly asking for data.

10 steps to modeling

We will suppose that the modeler knows the phenomena of interest, and has data or specific observations that need to be explained. A good modeling approach needs a good phenomenon to describe. Below we will highlight a modeling process that consists of 10 main steps, grouped into 4 sections (Figure 1): A. framing the question; B. implementing the model; C. model testing; and D. publishing the model. Throughout the discussion, we will use a common example 109 phenomenon that will be well known to most of our readers: Assume we did not have clocks and 110 ignored that they once existed. Now imagine what an archeologist finding a clock (such as the 111 one in Figure 2) would go through to find out what it was for. Similarly to the Antikythera 112 mechanism (https://en.wikipedia.org/wiki/Antikythera mechanism), we would have to build a 113 model to explain certain aspects of the observed clock behavior. As such, the clock device in 114 analogy to the brain computes something and we're trying to figure out what it is. We will discuss how we might model the the movement of "hands" across the markings of an analog 115 116 clock. We will go through all the modeling steps as part of a modeling process with which we 117 could address the movement of the digits of the clock. With this example we will be able to 118 highlight all the ten steps of the modeling process.



120 121

122

123 124

125



Figure 2: Mechanical watch. Even knowing what it does, it's inner workings are far from trivial. Imagine an archeologist finding one of those and not knowing what this is for.

A. Framing the question

Step 1: Finding a phenomenon and a question to ask about it

126 The starting point to all models is a question related to a phenomenon of interest. Thus, the 127 first practical step for the modeler is to build a list or table of the critical observations (e.g. see 128 Table 1) that define the interesting aspects of the phenomenon - what sets it apart from other 129 things or for which we lack good explanations. For the clock's movement the distinguishing 130 features for us are: they are precise, they are circularly periodic, and multiple hands have 131 nested periodicities. These periodicities are approximate multiples of another precisely timed 132 phenomena - the rotation of the earth. Defining the precise phenomenon is critical to asking a 133 good question.

134 135

Table 1: Example of critical, distinguishing observations of the clock.

Question- Answer Phenomenon	what	how	why
-----------------------------------	------	-----	-----

1 tick/sec	Loud for 100ms then silent for 900ms	Whatever that thingamajig is called	Timekeeping. Duh.
Gears exist	Notches on circles	More notches = slower rotation	Translate faster rotation / slower rotation
1 rot/hour	periodicity	120 notches on hour ring, 2 on second ring	Because an hour is a useful division of the day

148

158

137 Once we have characterized the phenomenon, we need to define a meaningful modeling 138 question. It helps to get clarity on the type of question we are asking: Are the characteristics that 139 define the what of the phenomena well-formed, or do we need to better describe the data? Do 140 we want to ask how something works? Or are we interested in why the phenomenon exists in 141 the first place? The observations about the clock can lead to different types of questions - what 142 is the relation between the gears and the movement of the arms, how do the gears produce the 143 observed pattern of arm motions, and why would anyone build such a mechanism (see table 1). 144 None of these questions is inherently more or less interesting; they could all represent legitimate 145 goals for a model builder (K. Kording, Blohm, Schrater, & Kay, 2018). However, a clear choice 146 of such a goal is essential to allow meaningful models. Clearly specify your goal at the outset, 147 to yourself and in all your communication about the modeling project.

149 Once we have a general phenomenon and question in mind, we can demarcate which 150 aspects of the data our model should capture. Without an exact question, chances are high that 151 one will get lost in the vast oceans of the unknown. This leads to our first dictum of modeling: 152 Write everything down in a precise way!, beginning with the question. Imprecise questions lead 153 to rapid failure. "Model a clock" would be a bad definition of a goal, after all it does not identify 154 key observations or criteria for success or failure. "How do the angles of the hands predict the 155 time of the day?", on the other hand, would be a well defined question. It both specifies the 156 phenomenon (time of day relation) and implies criteria for success (low variance at predicting 157 time of the day).

159 At this step, it is also helpful to identify aspects of observations that the model will not 160 address to answer the question. E.g. we may decide that we do not (for the current model) care 161 about the mechanisms in the clock. By focusing on distinguishing features of the phenomena 162 together with intuitions about the factors that should be included in an explanation, the model is 163 focused both towards a concrete question and at an appropriate level of abstraction. By 164 maintaining focus, we avoid the inevitable "mission creep" which results from having a fuzzy 165 question; fuzzy questions inevitably pull researchers towards attempting to answer a much 166 larger family of apparently related questions. Having focus also provides a natural Occam's 167 Razor quality to our models. Through focus our models address the knowledge gap central to 168 the question while minimizing the complexity of the approach.

170 As part of the objective, the model evaluation method must also be defined. This leads to 171 our second modeling dictum: "Know when to stop!". A well-defined modeling goal must have a 172 well-defined stopping criteria, or else we will suffer endless mission creep. We should be able 173 to answer the following questions: When are we satisfied with the new model? What would it 174 mean for a model to be better than another model for our criterion? These are difficult 175 questions, but there are clear desiderata that good evaluation criteria should adhere to. The 176 evaluation must ensure the model incorporates the critical observations. The evaluation must 177 make the model connect with actual or potential data. In the clock example, we might wish to 178 reproduce the observed periodicity with a low error. Or we might want to provide an explanation 179 of why there are so many gears and what they're good for. Thinking about a specific experiment 180 that could potentially answer the questions posed is often tremendously useful to ensure these 181 desiderata are satisfied. It provides a specific, tangible and intuitive instantiation of an abstract 182 question. Moreover, it inherently provides a benchmark goal for the model to be designed. 183 Indeed, the model should be able to simulate this exact experiment to provide a model-based 184 answer. In the clock, removing a gear or changing a gear ratio could be a good experiment to 185 test the role of gears. Being able to simulate results from a hypothetical experiment or real 186 experiment thus becomes part of the modeling objective. 187

188 Finally, it is also important to determine precise evaluation criteria based on well-defined 189 qualitative and/or quantitative properties the model should exhibit. This is crucial because data 190 derived from experimental observations is naturally variable and thus determining criteria which 191 allow us to judge the model's performance is important to ultimately determine when the 192 modeling exercise is accomplished. For example, is the goal to reproduce general trends / 193 tendencies, or is a detailed match of model and data of importance? Are there certain specific 194 experimental effects or relationships that the model must reproduce? How will performance be 195 measured? E.g. if the clock is really meant for timekeeping, then a model of the clock should 196 match its periodicity very closely (i.e. within measurement noise). We will further elaborate on 197 the model evaluation in section C (steps 8 and 9). Establishing the evaluation method right from 198 the start will ensure a fair, critical evaluation of the modeling effort and a timely finalization of the 199 model. 200

In our experience, step 1 is the most difficult for both novice and experienced researchers.
 It is the step that requires the most thought, and it is a step often revisited for refinement after
 realizing that the subsequent steps aren't working.

Step 2: Understanding the state-of-the-art

206 Before diving into the modeling itself, it is obviously essential to survey the literature. This 207 survey serves to provide additional information about the phenomena, if there is controversy or 208 specific conditions under which it occurs, and provides background on the set of questions that 209 have already been addressed. From a modeling perspective, it provides insight into the types of 210 abstractions and approaches that might have already been used: What has already been done 211 in terms of modeling? Are there previous models that one can use as a starting point? What 212 hypotheses have other researchers (theoreticians and experimentalists alike) emitted regarding 213 the phenomenon in question? Are there any alternative and/or complementary models or

240 241

explanations? In the clock example, we may know the elementary theory from school that if we have a gear with N cogs and another with K cogs that it translates the rotation speed as N/K. This second step will ensure that no important aspects (theoretical and experimental) related to the model are accidentally omitted. It will also provide the specific data sets and/or alternative models to compare the new model against. In addition, this review might provide insight as to the specific evaluation criteria (e.g. root mean square fit error) that are typically employed in the field. A literature survey should thus always be carried out prior to building a new model.

222 It is also important to gain an intuitive and practical understanding of previously proposed 223 models and theories. Such an understanding can only be obtained by re-implementing previous 224 models and explore their potential and limitations in a hands-on fashion. Exploring previous 225 models familiarizes the researcher with specific approaches, toolkits and mechanisms that have 226 previously been proposed. Exploring strengths and weaknesses of existing models will help 227 identify and justify the need for a new model. Step 2 can be characterized as a foraging task 228 where the researcher better characterizes the phenomena, the explanatory gap and gathers 229 together a set of possibly useful ingredients into the modeler's workshop: concepts, methods, 230 mechanisms etc.

232 Finally, the literature review should also allow determining the skill set needed in order to 233 understand previous modeling endeavors. This could result in the need to learn new skills, 234 whether or not those skills will also be helpful for building the new model. Thus, a good 235 understanding of the state-of-the-art of a field is instrumental to understanding previous models 236 and proposing a new model in the light of previous work. However, our question-centric 237 approach eschews premature adoption of any of these approaches. Instead we advocate 238 evaluating previous approaches through the lens of the focused question and its basic 239 ingredients.

Step 3: Determining the basic ingredients

242 After defining phenomena and objectives, we can now become a bit more specific. Every 243 modeling effort starts with an intuition that will provide an inventory of specific concepts and/or 244 interactions that need to be instantiated. What variables and/or parameters in the question and 245 inventory are needed in the model? Are those constants or do they change over space, time, 246 conditions etc.? Are there any concepts (e.g. value, utility, uncertainty, cost, salience, goals, 247 strategy, plant, dynamics, etc.) that need to be instantiated as variables? Can these variables 248 be observed / measured directly or are they latent (internal) variables in the model? In order to 249 instantiate latent variables, they should be related to potential measurements, whether 250 practically possible or not. In our clock example, the angular speed of the gears (latent variable) 251 might matter in determining the movement of the arms (observed) and we know it's constant for 252 a given gear but different across gears. What details can be omitted (e.g. materials the clock is 253 made of)? What are the constraints, initial conditions? How are these variables expected to 254 interact? For example, there is a specific relationship between gear speeds in the clock that is 255 constant and determined by the fraction of number of cogs. What should be the model's inputs 256 (potentially under experimental control) and outputs (that could be measured)? I.e. outputs 257 should typically be the same as the data the model addresses. Answering these questions will

273 274

set up the elements that are required in the model as well as the specific conditions that have tobe satisfied by the model.

261 A second much more difficult to acquire - but crucial - set of instruments for the modeler is a 262 library of potential explanatory mechanisms. Such a library is usually collected over time by 263 hands-on exploration of different models, approaches, pieces of math and algorithms. This goes hand in hand with building an intuition for a research field through exploratory data/model 264 265 analysis and careful reading of the relevant literature. An intuition is then formed as a result of 266 experiences with different model classes and data. For the clock example, models that produce 267 oscillatory behavior (i.e. periodicity) might be of particular interest. We claim that there is no way 268 around this learning by doing step (and regular cataloguing of this explanatory set should be a 269 priority for the community). But as a result, the potential required explanatory mechanism(s) will 270 also help in providing specific concepts and interactions that need to be instantiated. Once the 271 model ingredients and potential mechanisms have been identified, specific hypotheses can be 272 expressed in mathematical language.

Step 4: Formulating specific, mathematically defined hypotheses

275 Contrary to the question asked in Step 1, hypotheses propose a specific relationship that 276 could explain a given phenomenon. To formulate a hypothesis in modeling terms, we need to 277 map our intuitions and proposals about mechanisms and variables into precise mathematical 278 language. In this sense, a model is a mathematical quantification of verbal hypotheses. The 279 first step in achieving this is to relate the ingredients identified in step 3 by quantifying specific 280 hypotheses. For example the 60:1 ratio of periodicities between the smaller hands of the clock 281 corresponds to tracking seconds/minute. These hypotheses can be expressed in terms of 282 relations between variables and restate the original question from step 1 in the form of relations 283 between variables, mediated by hypothesized mechanisms and interactions. Thus, these 284 hypotheses are the ones that are identified from the original question and ask: What is the 285 model mechanism expected to do? How are the different parameters expected to influence the 286 model results? Answering these questions with words / sentences will set the modeler up to 287 start expressing relationships between parameters and variables in mathematical language. 288

289 Going back to our clock example and supposing we do not know what this device is for, a 290 series of hypotheses can be emitted related to the what, how and why questions. First, we can 291 hypothesize that the gears will lead to different arm speeds. Second, it is the exact gear ratio 292 that is of importance and this gear ratio is determined by the dynamics of the spring-balance 293 wheel system. Third, we can hypothesize that clocks are there as time keeping machines. For 294 all of these hypotheses we have made use of our inventory of observations about the movement 295 of the arms, the gears, the spring and the balance wheel. We also need to keep in mind the use 296 of the clock, i.e. people use the clock for scheduling purposes and to regulate / coordinate 297 human behavior. These verbal hypotheses represent the starting point for mathematical 298 abstraction, identifying key components and concepts needed for each question. 299

300 Once the hypotheses are spelled out, variable names should be assigned so that 301 hypotheses can be expressed succinctly in those terms. What mathematical relationships are 302 expected? It is good to be explicit here, e.g. y(t)=f(x(t),k) but z(t) does not influence y. Can we 303 hypothesize anything about the form of f? One advantage of this explicit mathematical notation 304 is that it is also made clear that x, y and z change over time while k is a constant. Constraints, 305 initial conditions and any other known or expected relationships can be expressed in a similar 306 way. In our clock example, we can first write that angular velocity of the slowest hand is vx = 307 f(r,v0), where r is the gear ratio and v0 is the resulting speed of the spring-balance wheel 308 system driving the gears (latent variable); we hypothesize f to be linear. We can further write a 309 relationship for the gear ratio r and hypothesize that the gear ratio between 2 arms determines 310 their relative angular speed. Let z(t) be the angle of the fastest hand, y(t) the intermediate, and 311 x(t) the slowest hand. Then we hypothesize $y = r^* y$, thus the angular position $y(t) = r^* y$ 312 $r^{*}x(t)$ +constant mod 2*pi, and as above, y(t) is not influenced by z(t), rather the converse is true. 313 The spring-balance wheel system should act like a harmonic oscillator determining v0, i.e. v0 =314 f(m,k) where m is the mass of the balance wheel and k is the spring constant. Formulating 315 hypotheses for the why question is also possible. If it is indeed a time-keeping machine used to 316 organize human activities (as opposed to a similar-looking astronomical position tracker such as 317 the Antikythera mechanism, for example), then there should be a correlation between different 318 peoples' behavior that is based on their consultation of the clock (and no correlation if it was an 319 astronomical or other device). In that case, we could write that the clock-based behavioral event 320 times Ti between different people should be highly correlated i.e. Ti = f(Tj) for j≠i, where f is 321 expected to be linear and of slope 1. The resulting mathematical relationships constitute the first 322 step of abstraction that will determine the model approach and identify the model ingredients 323 needed. In addition, these hypotheses will later be evaluated against model behavior. Lastly, 324 translating the specific hypotheses into mathematical language will ultimately also help in 325 "selling" the model to the research community. Indeed, the more precise the hypotheses, the 326 better the modeling approach can be justified. 327

Finally, it should be noted that Steps 1-4 are linear in an ideal case scenario, but often need to be carried out iteratively (see Figure 3). Indeed, every step has the potential to unmask a weak, imprecise, already answered, not interesting or too ambitious question. In that case the original question has to be modified, adapted, clarified or changed altogether, after which all following steps require re-consideration. This can also happen at later stages during the modeling exercise but if Steps 1-4 are carried out properly, this should be much less likely to happen. We are now at the point where the practical modeling can begin.

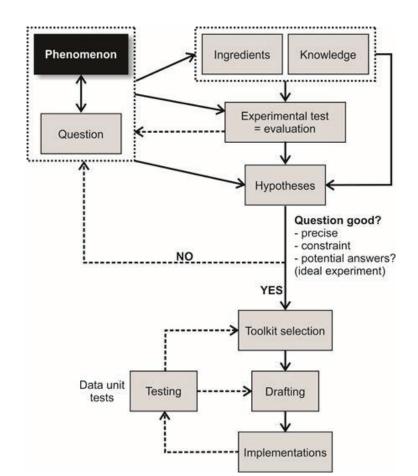


Figure 3: Iterative view of the first steps of the modeling exercise. Consecutive thought processes often identify lack, omissions, imprecisions and uncertainties that require the modeler to go back and refine the thoughts. This is true when framing the question and independently applies during model implementation. Note that these two processes are serial. One should not start the implementation process without having fully satisfied the all model framing criteria and steps. Solid arrows denote direct transitions/dependencies; dashed arrows stand for iterative reconsideration. Once a phenomenon/question is identified, required ingredients and literature review are carried out, which ideally leads to a potential experimental test. If no such test can be found, maybe the question needs reformulating. One should be able to identify specific hypotheses; otherwise there is a lack of specificity/precision in the question that needs to be

revisited. Toolkit selection, drafting and implementation of the model involves iterative unit testing. Unit testing can identify pitfalls in drafting or even in the choice of the toolkit (less frequently) that requires adjustment of the model plan.

B. Implementing the model

Step 5: Selecting the toolkit

353 Once the modeling goals are set and the hypotheses are quantified, the most appropriate 354 modeling approach to address the question needs to be selected. It is important to state that 355 different model toolkits can potentially provide an answer to the same question asked. But not 356 all toolkits are equivalent; guite the opposite. Indeed, different toolkits afford answering different 357 types of questions, such as being able to extrapolate versus finding mechanistic reasons for a 358 given phenomenon. Important considerations are: what modeling tools should be used (e.g. 359 mechanics) and what level of abstraction (e.g. what is the purpose of this device) is 360 appropriate? Based on the hypotheses and goals, this should now be relatively easy to do. In 361 the clock example, we might not care about the material properties of the gears but only the 362 number of teeth in the gears. We also cannot lump all gears together because they activate 363 different arms. As a general rule, the model should stay as high-level/abstract as possible, but 364 be as detailed as necessary (Occam's razor, (Feldman, 2016; Seiradakis & Edmunds, 2018)). 365 The choice of a modeling toolkit then allows the production of a real model. 366

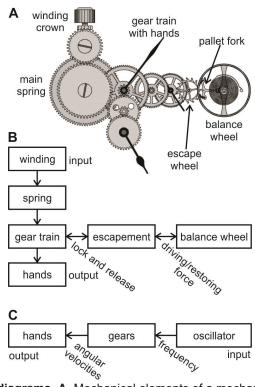
367 Determining which toolkit to use can be far from trivial and requires prior knowledge about 368 the toolkit. As a guideline, a good question to ask is how flexible the toolkit is in terms of 369 behavior. There is no "right" tool and often there is more than one option to choose from. Tools 370 should interface with data that the model is trying to address. For example if data consist of 371 changing time series then the toolkit has to have a dynamic component that can reproduce 372 those time-dependent signal changes. If we're interested in understanding the spring-balance 373 wheel and gear mechanism of the clock, we might turn toward mechanical finite element toolkits 374 to understand the physical properties of these elements influence the functioning of the clock; or 375 we could just care about the resulting clock arm dynamics and use higher-level kinematics tools 376 instead. Toolkit selection supposes a good knowledge of what the strengths and limitations of 377 each available toolkit are. Preference should be given to toolkits that have more flexibility, span 378 a wider range of behaviors, and are potentially lumpable (i.e. can be reduced in size by using 379 techniques such as population averaging or state-space reductions). E.g. neural networks span 380 a large range of behaviors but lumping is hard. On the other hand linear systems theory lumps 381 well but does not have the same level of detail as neural networks (but see (Eliasmith & 382 Anderson, 2004) for one particular way to do that). In summary: Knowledge is key. 383

384 Choosing the toolkit also means determining how the model will be solved (i.e. simulated). 385 For example, can an analytical solution be computed or is numerical integration of equations 386 required? If numerical integration is needed, what is the temporal, spatial, etc. resolution? In the 387 eye movement literature many models make use of the Laplace transform of dynamical 388 systems; this would require learning about the Laplace formalism and how to use it. Here, we 389 will assume that a way to solve the equations of the chosen toolkit can be found. This requires 390 of course knowledge about the appropriate fields of physics, mathematics, computer science 391 etc. if applicable, and it is very difficult to succeed as a modeler without such appropriate 392 background.

393 394

Step 6: Planning the model

We are now ready to start building up the model. This is the point where diagrams are drawn, sketches can be made, equations are formalized and preliminary pieces of code are 397 written. The goal of this step is to put all the components of the hypothesized relationships and 398 explanations in place. As the most important rule the model should always be kept as simple as possible! It is advised to start with a first draft of the model on paper. All toolkits allow for a 399 400 graphical representation to be built, but the nature of these drawing can be guite different. For 401 example, a mechanical model of the clock (Figure 4A,B) will look different from a dynamical 402 systems description (Figure 4C) of the clock movements, including potentially different inputs 403 (such as in Figure 4), latent variables, constants, initial conditions and outputs. Draw out the 404 model components and how they connect to each other / influence one another. This flow 405 diagram (e.g. Figure 4B,C) will help organizing the equations. It will allow to explicitly indicate 406 which variables "flow" from one model component to the next. This model diagram will set up 407 the basic components that are expected to be required in the model. 408



eNeuro Accepted Manuscript

409 410

411

412

413

414

Figure 4: model diagrams. A. Mechanical elements of a mechanical clock. B. Flow diagram equivalent of the mechanical clock. C. Dynamical systems equivalent of the mechanical clock. Note: inputs are different between full model (B) and reduced model (C). Exemplary variables (tilted text) passed between model elements also differ in nature.

415 Now each model box, icon or flow can be considered individually and its internal workings 416 should be drafted in terms of mathematical equations. These should be explicit equations that 417 can later be implemented in simulation programs. In case of the clock example, the gear train 418 box might be subdivided into one functional box for each gear in the flow diagram determining

437

452

460 461

419 the equations of motion of the gear and relating the boxes' input (previous gear's angular 420 velocity) to the output (this gear's angular velocity). It can require extensive work to identify the 421 appropriate mathematical relationships, equations and formalisms. But at this stage, filling the 422 boxes, quantifying the icons and/or specifying the interactions between them should be 423 relatively easy since the basic input and output variables of the model's subsystems have 424 already been defined and the modeler's goal thus "only" is to relate those variables. It is 425 important to keep in mind here that a model must include a way to relate model variables to 426 measurements. Otherwise the modeling exercise will typically feel pointless. Ultimately, the 427 drafting process will result in a first model on paper that is ready to be implemented and might 428 become a model diagram in a subsequent publication.

Step 7: Implementing the model

The model is now ready to be implemented. This means that computer simulations can be set up and run and/or analytical solutions can be found. Each box, icon or flow relationship identified in Step 6 should be implemented separately and tested or understood individually before connecting them into the overall model. This "unit test" procedure will ensure the individual components' functionality before evaluating the more complex behavior of the full model.

438 Individual model components can then be combined. If there are any alternatives or 439 uncertainties, it is advised to start with the easiest implementation of the model or of part of the 440 model and test its functionality along the way. A general guideline is to build up the model step 441 by step and test its function at each step. Starting with a simple version of the model and 442 progressively adding all the elements, will not only produce an understanding of what simpler 443 models can do but also minimize errors in construction. Moreover, playing with all the 444 components of the model on implementation time can provide deep insights into the way they 445 actually work. In our clock example, there are gears for rewinding the clock's spring 446 mechanisms. Those gears can be modelled but they will not influence the arms movement 447 (unless the spring is loose of course). Thus these rewinding gears are not crucial for the 448 timekeeping function of the clock mechanism and can be left out if that kind of understanding is our goal. Answering the question why a certain model component is crucially needed will 449 450 ultimately allow justifying the model architecture during the publication process. This process 451 should be continued until the model has been fully implemented.

Once we have implemented a model we want to make sure we properly understand our own implementation. This makes it necessary to deeply analyze its behaviour (Otto & Day, 2011). We should plot model behavior as a function of model parameters. We can analyze model stability / equilibrium points. We can ask how similar the model performs to known models, e.g. those that can be analytically solved. Each modeling toolkit usually comes hand in hand with a set of model analysis tools; details about the latter can be found in the specific toolkit literature. All these steps may help us in finding mistakes in our model implementation.

C. Model testing

462 Step 8: Completing the model

eNeuro Accepted Manuscript

472

480 481

489

499

505

463 One of the hardest questions in modeling is to decide when to stop improving the model and 464 call it final. Referring back to the goals (step 1) and hypotheses (step 4) is crucial here. Does 465 the model answer the original question sufficiently, i.e. with enough detail to advance knowledge 466 in the field of study? Equally importantly, does the model satisfy the evaluation criteria that have 467 been determined prior to building the model? Does it speak to the hypotheses, either confirming 468 or invalidating them? In other words, can the model produce the parametric relationships that 469 have hypothesized in step 4? If the answer to all these questions is "yes", then the modeling 470 exercise might be done. If the original goal has not been met, then the modeler may need to get back to the drawing board. 471

We need to be mindful on finishing a project when the time has come. On the one hand we can usually improve model fits, on the other hand, we do that at the risk of overfitting the data we have. Occam's razor might help here to determine if it is worth considering more complicated models with more parameters, that are perhaps irrelevant or uninterpretable in order to obtain a better fit to the data. The cost of such more complicated models is always the reduced explanatory power. This is mathematically quantified in measures such as the Akaike information criterion, as explained in the following step.

Step 9: Testing and evaluating the model

In steps one and four, we set up goals/ hypotheses and objectives for our modeling approach. Once we have implemented and tested the model we can now evaluate how well we did in the modeling approach. How to evaluate how well a model did, supremely depends on the nature of the goals. For example, if we only care about the relation between the second and the minute digit of the clock, then explaining their relative movement well would be sufficient. If we want to answer why clocks exist, our answer would have to look very differently. The objectives we defined further up determine how exactly a model is to be evaluated.

490 However, many different modeling approaches are aimed at describing data. This generally 491 leads to a statistical problem - how can we ask which model better describes the data. Statistics 492 has given us many tools to ask this guestion. These range from the mean squared error, to 493 methods that correct for the number of free parameters (e.g. the Akaike information criterion) to 494 the ability to predict new and unseen data. Model comparison is a centerpiece in the modern 495 modeling enterprise. Indeed, model comparison is useful to compare a new model against 496 existing precursors / alternatives. It is also often useful to build a class of models instead of just 497 creating one specific instance, in which case model comparison is often used as a means of 498 selecting the best model among the class of models proposed.

500 Finally, it is important to ask questions about generalizability. The model explains the 501 phenomenon we set out to describe. But knowing this is not enough. Will the model also 502 adequately describe similar situations? Can what we learned from one clock generalize to 503 others? Without quantifying generalization it is unclear how valuable a model is and no 504 modeling study should be finished without asking the generalization question.

Debunking myths:

- Models are not built to win a beauty contest but to explore the unknown.
- Modeling is not a grade school art show: Multiplying evaluation criteria to find one in which your model succeeds is not a good idea.
- The model that best fits your data may not be the best model (e.g. because of overfitting and limits to your data)
- Modeling is not a fashion show: models should not be judged in terms of fashionable concepts and mechanisms.
- Models are not your children. Even if you have created them, diapered them, trained them, etc., don't be a parent protecting your model at all costs but accept if they fail. After all, it's meant to fail! The question is how much can we learn from it and how much can it advance knowledge until it fails.
- Don't be a model bigot. You shouldn't just hate a model because it uses a different language than you would use. Understand what they say first! Irrational toolkit preference is inappropriate and hinders knowledge advancement. Don't judge the mechanic by its toolkit but by what (s)he can do with it!

D. Publishing

506 507

508

515

Step 10: Publishing models

509 Once everything has been done right, the model has been built, simulations are running and 510 satisfactory results have been obtained, the goal is to communicate those findings through a 511 scientific publication. This is a tricky exercise in itself and it is worth spending a few words 512 highlighting aspects that will much improve the likelihood of acceptance. In addition, this section 513 should be a guideline equally for authors and reviewers so that model evaluations can be as fair 514 as possible.

516 Model publishing essentially comes down to conveying each of the previous 9 steps to the 517 audience in a structured fashion (K. P. Kording & Mensh, 2016; Otto & Day, 2011). The 518 introduction section should describe the phenomenon / question that the model addresses (step 519 1), provide relevant background information from the literature review (step 2) and maybe 520 introduce some of the ingredients needed (step 3) as well general hypotheses (step 4). Methods 521 will detail all model ingredients (step 3) and hypotheses (step 4), justify the choice of the toolkit 522 (step 5) to answer the question asked and meet the goals. The final graphical draft of the model 523 (step 6) typically becomes the first figure. Implementation details (step 7) as well as the 524 procedures of model testing and evaluation (step 9) will also be detailed in the Methods section. 525 Results will summarize model performance (step 8) and provide the testing and evaluation 526 statistics (step 9) along with answering the original question (step 1) and speaking to each of 527 the specific and general hypotheses (step 4). Thus overall, following the 10 steps of modeling 528 also streamlines and simplifies the publishing step, especially if detailed notes have been taken 529 all along the way.

531 Finally, there are a series of important guidelines to respect when publishing models:

- Know the target audience. Write in a way that your audience can understand. In most cases the target audience should be experimentalists!
- In order for a model to receive the appropriate appreciation, it is absolutely crucial to clearly describe what the goals, hypotheses and performance criteria were (K. Kording et al., 2018)!
- A model should always be graphically represented (Rougier, Droettboom, & Bourne, 2014) if at all possible.
- Show model behavior in parallel (i.e. side by side or superimposed) with the data that the model was designed to explain. This is a powerful way to prove to the research community that the model mechanisms have been correctly interfaced with the produced behavior.
- Publish the model code. Clean up the code and make it readable and understandable to others. Ideally, the published code should reproduce all results figures in the article. Publishing the code hugely increases the usefulness of the model for science (Prlić & Procter, 2012). Consider ModelDB (<u>https://senselab.med.yale.edu/modeldb/</u>) or similar repositories to publish your model.
 - Publish the data that you fit your model to in one of the relevant databases (e.g. crcns.org, figshare, OSF.io, etc).

Discussion

553 We have argued that following these 10 simple steps should leave modelers with little room 554 for failure. As mentioned before, we have successfully applied this pipeline 2-week long small-555 group model building exercises at CoSMo. It is worth pointing out that this success was 556 irrespective of model type or class, i.e. it worked for models ranging from neural networks to first principle derivations of normative behavior, and from model-driven data analysis to pure theory. 557 558 Of course, for each type of question/model, the extent and practical implementation of the 559 different how-to-model steps might looks different and be more or less extensive. However, 560 importantly all step tend to apply to all types of modeling approaches.

What's a good model?

563 Consider you have done everything right as outlined in the 10 easy steps to modeling. You 564 framed the question precisely, had specific testable hypotheses, choose the right toolkit, 565 implemented the model, fit it to data, selected the right number of parameters / the best model, 566 cross-validated your results and compared your best model to alternatives from the literature. 567 Does that mean your model is a good model? In fact, what are the criteria of a good model?

568

561 562

532

533

534

535

536

537

538

539

540

541

542

543

544

545

546

547

548

549

550

551 552

> "All models are wrong, but some are useful" (Box, 1976) "The words true model represent an oxymoron" (Anderson & Burnham, 2002) "Everything should be made as simple as possible, but no simpler" (Einstein)

595 596

570 There are many potential criteria motivating the development of a model and many of them 571 are valid criteria in judging whether the goals have been achieved (K. Kording et al., 2018). 572 Criteria could be: explain data, interface with data, generalizes within sample / out of sample, 573 robustness, reproducibility, bridging fields, across-fields predictions, interpretability, inspires 574 experiments, clinical relevance, falsifiability, mechanistic insight, people care (funding), new 575 predictions, technological applications, intervention / policy implications, non-arbitrary structure 576 (elegance), subsumes previous models / data (unification), self-consistency, plausibility of 577 hypotheses, simplicity, computing efficiency, realism, and normativity. Evidently, not all models 578 satisfy all those criteria; in fact, satisfaction of any single criterion might be sufficient to consider 579 the model as being of value. The precise choice of the evaluation criterion should be dictated by 580 the modelers stated goals and the field's consensus on how to evaluate of such goal is met. 581 However, one universally important aspect about modeling is the subsumption principle, i.e. a 582 good model should capture all existing phenomena in a domain, not just the data in front of the 583 modeler.

585 Depending on the model criteria (see above), questions and goals, a different model toolkit 586 might be chosen for the same phenomenon to explain. This is because different toolkits allow 587 answering different types of questions and achieving different modeling goals (Kording et al., 588 2018; Blohm et al., 2019). As a result, models vary greatly along many dimensions, such as 589 granularity (David Marr's computational, algorithmic and physical/implementation levels), 590 generality (Peter Dayan's and Larry Abbott's descriptive, mechanistic and interpretive models) 591 or scale (physical extent of system modeled). Depending on where is a model is situated in this 592 high-dimensional model space, there are typically different constraints, scopes, evaluation 593 criteria, etc. for a model. It is thus useful to know where a model is situated in this space as it 594 constrains the goals and defines the limitation of a model (Blohm et al., 2019).

Good modeling practices

597 Meaningful model development in neuroscience should go hand in hand with good modeling 598 practices. For example, iteratively modifying the model structure to obtain a better fit to the data 599 is often done; however, this is not always advised because changing the model structure might 600 imply changing the hypotheses on the fly, which is essentially HARKing ("Hypothesis After 601 Results" justification). Furthermore, pre-registration might prevent some of the biases in model 602 comparison that stem from researchers' motivation to show that their new model fits data better 603 than previous models. Following our 10 simple rules in the correct order (see Figure 3) guards 604 against this (often involuntary) fallacy. We strongly advise to not make any changes to the 605 model hypotheses and structure after steps 1-6 have been completed. One good way to stay 606 honest would be to pre-register (Nosek, Ebersole, DeHaven, & Mellor, 2018) the model plan, 607 outlining hypotheses and test strategies developed in steps 1-6. This does not prevent 608 researchers from performing crucial adjustments to their models if initially hypothesized models 609 fail to produce the expected result. Crucially though, pre-registration "forces" authors to report 610 the iterative adjustments, allowing the community to benefit from the insights gained throughout 611 the process. For example, one could imagine a situation under which the clock's hypothesized 612 purpose would be to predict the movement of the stars; knowing this is wrong would help the community move forward in understanding the clock. Note, that pre-registering the modeling 613

study in itself is to be considered separately from pre-registering potential experimental
predictions that result from the model. In summary, we suggest that pre-registration of modeling
efforts would lead to a cleaner, more comprehensive and reproducible model building process in
which logical steps and reasonings are clearly outlined and reproducible.

There might be limitations to when a modeling study should be pre-registered. The above procedure might be most suitable when a model is a specific implementation of a hypothesized mechanism to explain previously described phenomena for which there is data. It might make less sense when the modeling effort consists in developing new theoretical tools or general theories, e.g. a new machine learning approach or a new principled way of learning. However, we would argue that these exceptions are rather rare in neuroscience research compared to the abundance of models that directly target data.

Conclusion

628 This 10 step pipeline has proven to remove some of the apparent arbitrariness of the 629 neuroscientific modeling process and provide teachable instructions on how to succeed in 630 modeling. Indeed, currently modeling looks much like a fashion show with the whims and trends 631 dictating what's hot. This arbitrariness in the modeling approach may also lead to misguided 632 model judgments. We emphasize that modeling is not a beauty contest; models need to be 633 judged based on their well-defined goals, not their appearance or fashionability. To allow of fair 634 judgment, authors have the responsibility to clearly lay out their thought process. While this 10 635 step guide is tailored toward the neuroscience community, it should help achieve this goal 636 throughout life sciences and beyond.

637 638

618

626 627

Example box

Modelling eye movements

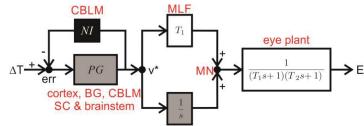
David A. Robinson is generally considered the father of quantitative oculomotor research. Here, we will use one of his most influential modeling studies (Robinson, 1973: Models of the saccadic eye movement control system) as an example illustrating our 10 steps "how to model".

- *Step 1*: In general, Robinson asked whether we can understand the neural organization that controls saccadic eye movements by establishing relationships between computations in an abstract controller and the activity in subcortical brain areas, such as motor nuclei. In doing so, he is really addressing two different questions: (1) are eye movements expressible as the result of an abstract controller (causal question) and (2) is the neural activity compatible with latent variables in an abstract controller (explanatory question)? For the latter, Robinson referred to novel specific data from oculomotor nuclei.

- *Step 2*: Robinson grounds his model in the literature, using a previously published and highly influential model of the extraocular muscle and eye ball mechanics (Robinson, 1964) as a starting point for his oculomotor controller. He could also rely on electrophysiological recordings in oculomotor neurons (e.g. (Robinson, 1964, 1970)) as well indirect evidence for a neural integrator in the eye premotor circuitry (Skavenski &

Robinson, 1973). Finally, he obtained crucial intuitive insight from the stereotypical nature of saccadic eye movements, specifically the high degree of regularity of their velocity profile.

- *Step 3*: Since Robinson was interested in producing eye movements to a target, model input is an abstract motor goal and model output is eye position (see Figure Box 1). How did Robinson choose the right variables? How did he make sure that these variables were compatible with the phenomenology in terms of magnitude, resolution (level of detail) and timescale? It was known from oculomotor neuron electrophysiology that the eye plant needed a pulse and a step command to overcome elastic and viscous forces respectively. Robinson's model needed to generate such neural commands as latent variables and used a neural integrator to produce the step from a pulse. Finally, he needed a pulse generator that was able to convert a motor error (or goal) into a pulse that could then drive the saccade. He was only interested in reproducing average population firing rates, not single action potentials. He also only considered eye movements starting from the primary eye position.



MVN & NPH

Figure Box 1: Updated version of Robinson's simple saccade model (Scudder, 1988). Saccade target shift (ΔT) is compared to an internal estimate of saccade progression computed through the resettable neural integrator (NI, suggested by Scudder (1988), not Robinson) to provide a motor error (err). Based on circumstantial evidence, Robinson's insight led him to postulate the pulse generator (PG) to provide a desired eye velocity drive (v*). This pulse command was scaled to match the eye plant dynamics (gain T_1) and provided the saccade drive. However, Robinson recognized that visco-elastic forces would pull the eye back to primary position if not actively compensated for. This is how he

proposed the neural integrator (*s*) to provide a tonic drive that overcomes the viscoelastic forces. Tonic and phasic drives add up and are sent to extraocular muscles of the eye plant that he modeled as a second-order system to move the eye (E). Red labels are mappings of individual computations to specific brain areas. CBLM: cerebellum; BG: basal ganglia; SC: superior colliculus; MLF: medial longitudinal fasciculus; MVN: medial vestibular nucleus; NPH: nucleus prepositus hypoglossi; MN: motor neurons. Grey boxes indicate Robinson's innovations. Black box denotes a later modification of Robinson's model by Scudder (1988), included here for correctness.

- Step 4: Robinson hypothesized that saccades result from a pulse input to the ocular

plant. He also hypothesized that a neural integrator existed and that it integrated a scaled version of the pulse command. Pulse and step commands should then be added up again at the level of the motor neurons (see Figure Box 1).

- *Step 5*: Robinson used linear control systems theory as a toolkit to address his question because he believed that the brain needed to implement some natural neural control law and he knew that any such dynamical system could be locally well approximated by a linear system (see goals, step 1). In choosing this toolkit, he hoped to span all 3 levels of Marr from computational (i.e. overall system behavior describing eye movements) to algorithmic (how this behavior could be implemented most efficiently) to physical (neural population coding of the individual components of his model).

- **Step 6**: Robinson drew a draft diagram of the model given his knowledge and hypotheses (similar to Figure Box 1). He could then fill in the boxes using linear control theory language. For example, his hypotheses allowed him to write down a potential premotor circuit transfer function. He also already knew the transfer function of the eye plant from his previous work. Finally, he needed a pulse generator. Since little was known about it, he chose what he thought was the simplest arrangement reproducing the correct saccade dynamics. Note that Robinson also chose all his latent variables in his model to represent observable firing rates of real neural areas.

- *Step* 7: Robinson's first model was elegant in that it used known physiology to produce saccadic eye movements in a seemingly simple fashion. However, he knew that this model was unlikely to be able to reproduce other aspects of saccades or their neural control, such as saccades to moving targets. He (and other authors) therefore incrementally expanded his model in follow-up studies to include missing aspects.

- **Step 8**: Robinson considered his task achieved when his models were able to qualitatively reproduce the specific data he set out to model. He thereby answered his 2 initial questions, i.e. that latent variables in his model are indeed consistent with oculomotor electrophysiology and that linear control systems theory could accurately capture the brain's control of eye movements, at least in the brainstem.

- **Step 9**: Robinson only carried out qualitative model evaluations. This included comparing model and real eye movement behavior as well as comparing model predictions of latent variables to neuronal recordings. Nowadays, reviewers would probably encourage him to provide more quantitative comparisons with eye movement data as well as a critical evaluation of his models with other existing ones, but scientific standards were different in 1973. However, his model made very interesting predictions regarding the presence of a common neural integrator for all eye movements as well as a phasic (pulse) motor command. Since Robinson's eye plant model in 1964, he also believed that principles of linear control theory can be used to describe all eye movements, which led to half a century of extremely fruitful theoretical and experimental work (breadth of application). As a result of his model-driven approach, the eye movement system is now arguably the best understood neural system.

- **Step 10**: Robinson published his manuscript in a journal called Kybernetik (nowadays Biological Cybernetics), which is mostly targeted towards engineers trying to understand biological systems. He clearly laid out his goals, described all details of his approach and

relates his findings to experimental data. But enough said; we encourage the reader to generate his/her own opinion by reading Robinson's paper.

639 **References**

646

647

652

653

654

655

656

657

658

663

664

- Anderson, D. R., & Burnham, K. P. (2002). Avoiding Pitfalls When Using Information-Theoretic
 Methods. *The Journal of Wildlife Management*, 66(3), 912.
- Blohm, G., Kording, K.P., Schrater, P.R. (2019). Modeling in Neuroscience as a Decision
 Process. https://DOI.org/10.17605/OSF.IO/W56VT
- Box, G. E. P. (1976). Science and Statistics. *Journal of the American Statistical Association*,
 71(356), 791.
 - Eliasmith, C., & Anderson, C. H. (2004). Neural Engineering: Computation, Representation, and Dynamics in Neurobiological Systems. MIT Press.
- Feldman, J. (2016). The simplicity principle in perception and cognition. *Wiley Interdisciplinary Reviews. Cognitive Science*, 7(5), 330–340.
- Kording, K., Blohm, G., Schrater, P., & Kay, K. (2018). Appreciating diversity of goals in
 computational neuroscience. https://doi.org/10.31219/osf.io/3vy69
 - Kording, K. P., & Mensh, B. (2016). Ten simple rules for structuring papers. https://doi.org/10.1101/088278
 - Nosek, B. A., Ebersole, C. R., DeHaven, A. C., & Mellor, D. T. (2018). The preregistration revolution. *Proceedings of the National Academy of Sciences of the United States of America*, 115(11), 2600–2606.
 - Otto, S. P., & Day, T. (2011). A Biologist's Guide to Mathematical Modeling in Ecology and Evolution. Princeton University Press.
- Prlić, A., & Procter, J. B. (2012). Ten simple rules for the open development of scientific
 software. *PLoS Computational Biology*, *8*(12), e1002802.
- Robinson, D. A. (1964). THE MECHANICS OF HUMAN SACCADIC EYE MOVEMENT. *The Journal of Physiology*, 174, 245–264.
 - Robinson, D. A. (1970). Oculomotor unit behavior in the monkey. *Journal of Neurophysiology*, 33(3), 393–403.
- Robinson, D. A. (1973). Models of the saccadic eye movement control system. *Kybernetik*,
 14(2), 71–83.
- Rougier, N. P., Droettboom, M., & Bourne, P. E. (2014). Ten simple rules for better figures.
 PLoS Computational Biology, *10*(9), e1003833.
- Scudder, C. A. (1988). A new local feedback model of the saccadic burst generator. *Journal of Neurophysiology*, *59*(5), 1455–1475.
- Seiradakis, J. H., & Edmunds, M. G. (2018). Our current knowledge of the Antikythera
 Mechanism. *Nature Astronomy*, 2(1), 35–42.
- Skavenski, A. A., & Robinson, D. A. (1973). Role of abducens neurons in vestibuloocular reflex.
 Journal of Neurophysiology, *36*(4), 724–738.